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We study dynamical evolution of boson stars in general relativity.

I. MOTIVATION FOR STUDYING BOSON STAR: LITERATURE SURVEY

The study of boson stars find its motivation from various angels: For example they provide a good model to test theory of General Relativity. It's a good model (singularity-free) to learn the nature of a **strong** gravitational fields.(no one has ever done serious **dynamic** studies with boson star: test of GR against other graviational theories such as Scalar-Tensor theory.

In a more physically relevant contexts, scalar fields have been suggested in various contexts. For example, inflaionary theory proposes scalar fields as a DM candidate, electroweak theory proposes Higgs boson. In Kaluza-Klein and Superstring theories, scalar fields are fundamental fields which appear in a natural way after dimensional reduction.

Boson star: complex scalar field can form stable configuration having negative binding energy.— $E_{bd} = M - Nm$?

Global U(1) gauge symmetry — charge/total number of particles conserved.

For small perturbation: there is characteristic oscillation frequency: e.g. see Seidel and Suen, PRD 42, fig6(oscillation frequency of star vs total mass) $\omega \sim M^2$ at the Newtonian limit and curve reaches the top (ω_{max}) and drops.

number of particle N is usually astronomically large.

equilibrium state of boson star: balance between kinetic energy(K.E.) and gravitational energy(G.E.)

K.E. $\sim p \sim \frac{1}{\lambda}$ per particle. total K.E. $\sim N \frac{1}{\lambda} \sim \frac{N}{R} \sim mN$.

G.E. $\sim \frac{GM^2}{R}$

K.E \sim G.E. implies $GM^2 = N$ where $N \sim \frac{M}{m}$. So $M \sim \frac{1}{mG}$ and $G \sim M_{PI}^{-2}$. So $M \sim \frac{M_{PI}^2}{m}$ for non- ϕ^4 type coupling. astrophysical:

??

compton wavelength of a free boson $\sim m^{-1}$ from $\frac{\hbar}{mc}$. With self-interaction $M \sim \sqrt{\lambda} \frac{m_{PI}^2}{m}$ and w/o it $M \sim \frac{M_{PI}^2}{m}$ and $N \sim \frac{M_{PI}^2}{m^2}$. With self-interaction for $m \sim 1$ Gev, $M \sim M_{sun}$. this boson star is a possible candidate for non-baryonic dark matter, possibly detectable by microlensing experiment.

boson star is a candidate for dark matter that could have been created during a phase transition.

Galactic halo itself a condensed bosonic objects.(??)

Mass of boson star could be as big as solar mass, but their size would be certainly as large as 10s or 100s km !!

OK, I am very unorganized so far. From Kip. Thorne's article **gr-gc/9706079**, it says that when WD, NS or small BH spirals into a much more massive, compact central body, the inspiral waves will carry a "map" of the massive body's external spacetime geometry. Since body's spacetime geometry is uniquely characterized by the values of the body's multiple moments, we can way equivalently that the inspiral waves carry, encoded in themselves, the values of all the body's multipole moments. By measuring the inspiral waveforms and extracting their map, we can determine whether the massive central body is a black hole or some other kind of exotic compact object. If the measured moments satisfy the black-hole "no-hair" theorem, i.e., if they are all determined by the measured mass and spin, then we can be sure the central body is a black hole. If they violate the "no-hair" theorem (BTW how could we be so sure?) then either the central body was an exotic object— e.g. a spinning boson star— etc..

For boson star, stress energy tensor is anisotropic. Therefore the concept of an equation of state is completely inappropriate.

How to compute total energy instead of total mass(I have been computing total mass all along)? $E = T + V$.

from note.tex

Sun. MAY 11, 1997

Matt suggested to look at the problem of relativistic boson star binary. It's good place to come back to GR, brush my old knowledg of GR. As usual, the first problem to look at is in the context of spherically symmetric boson star.

Now, go back to Matt's thesis for spherically symmetric case and Robert's thesis for 3D case. Survey of relativistic boson star.

KG equation:

$$\phi_{;\nu}^{\nu} - m^2 \phi = 0 \quad (1)$$

Introduce auxiliary variables such that $\alpha_{,t}$ and $\beta_{,t}^i$ do not appear in the KG equation.

Start with spherically symmetric ground state problems. Matt's diss, SS-PRD42 paper, Colpi et al paper.

Slicing condition: Polar slicing and maximal slicing

Might want to try PN computation for boson star. – previous PN study of boson star? Want/Need to know EOM for each potential $U, V_j, W_j, PHI_1, PHI_2, PHI_3, PHI_4$ Need to know EOM for scalar field. D. Lai, PRL 76, 4878(1996)

Neutron Stars: the existence of an absolute mass limit is truly a GR phenomena: pressure, which supports the star against gravity, also acts as a source of gravitation.

NS binary merger: (1) source of gravitational wave (2) cosmological gamma-ray bursts

PN approximation: the inspiral at larger orbital radius may be treated by PN approximation

Question: how tidal field modifies the mass limit and central density limit of a compact object

Will, Clifford M. /Theory and experiment in gravitational physics./Rev. ed. 1993

QC 178 W47 1993

Will, Clifford M. /Was Einstein right? : putting general relativity to the test. / 1986

QC 173.6 W55 1986

Study of PN approx. MTW ch 39; This post-Newtonian approximation can then be used to calculate general relativistic corrections to such phenomena as the structure and stability of stars.

Most general spherically symmetric metric can be written by

$$ds^2 = (-\alpha^2 + a^2 \beta^2) dt^2 + 2a^2 \beta dt dr + a^2 dr^2 + b^2 r^2 d\Omega^2 \quad (2)$$

Choosing radial condition($b = 1$) which renders r direct geometric meaning($4\pi r$ = proper surface area for the sphere of radius r), polar slicing condition($K = K_r^r$) which implies $K_\theta^\theta = 0$ and zero shift which is actually implied by polar slicing(MattDiss p110), we get

$$ds^2 = -\alpha^2 dt^2 + a^2 dr^2 + r^2 d\Omega^2 \quad (3)$$

(we define mass aspect function to signal the formation of black holes, $M(r) = \frac{1}{2}r(1 - a^{-2})$. Derivative of the mass aspect is given by.)

Eqns to solve are (3.5) in Seidel and Suen PRD paper.

Reference

Matt/Robertdiss,RELBOSpapers

II. COORDINATE CONDITIONS

gr-qc/9808024 Kip S. Thorne: Ideas on how to set coordinate conditions to keep the coordinates co-rotating, here he proposed a testbed problem to test coordinate conditions suggested as co-rotating. His model problem is somewhat like binary neutron star system. He propose “minimal-strain” lapse and shift equations.

III. EQUATIONS, ETC.

Thu. MAY 29, 1997

(NEW) to compare with Fig.2 of SS PRD paper, we need to compute $g_{tt} = -\alpha^2$, $g_{rr} = a^2$, $\rho = -T_t^t$.

Note here that energy density ρ in GR is different from Newtonian case. Of course!

The scalar field Lagrangian is

$$L = -\frac{1}{2}g^{\mu\nu}\phi_{;\mu}^*\phi_{;\nu} - \frac{1}{2}m^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4 \quad (4)$$

which implies an energy-momentum tensor

$$T_\nu^\mu = \frac{1}{2}g^{\mu\sigma}(\phi_{;\sigma}^*\phi_{;\nu} + \phi_{;\sigma}\phi_{;\nu}^*) - \frac{1}{2}\delta_\nu^\mu(g^{\lambda\sigma}\phi_{;\lambda}^*\phi_{;\sigma} + m^2|\phi|^2 + \frac{1}{2}\lambda|\phi|^4) \quad (5)$$

For $\lambda = 0$, we get for - T_0^0 ,

$$T_0^0 = \frac{1}{2}g^{0\sigma}(\phi_{;\sigma}^*\phi_{;0} + \phi_{;\sigma}\phi_{;0}^*) - \frac{1}{2}\delta_0^0(g^{\lambda\sigma}\phi_{;\lambda}^*\phi_{;\sigma} + m^2|\phi|^2 + \frac{1}{2}\lambda|\phi|^4) \quad (6)$$

$$= g^{00}(\phi_{;0}^*\phi_{;0} + \phi_{;0}\phi_{;0}^*) - \frac{1}{2}(g^{\lambda\sigma}\phi_{;\lambda}^*\phi_{;\sigma} + m^2|\phi|^2) \quad (7)$$

$$= g^{00}(\phi_{;0}^*\phi_{;0} + \phi_{;0}\phi_{;0}^*) - \frac{1}{2}(g^{00}\phi_{;0}^*\phi_{;0} + g^{rr}\phi_{;r}^*\phi_{;r} + m^2|\phi|^2) \quad (8)$$

$$= \frac{1}{2}(g^{00}\phi_{;0}^*\phi_{;0} - g^{rr}\phi_{;r}^*\phi_{;r} - m^2|\phi|^2) \quad (9)$$

$$\rho = -T_0^0 = \frac{1}{2}(\alpha^{-2}\phi_{;t}^*\phi_{;t} + \alpha^{-2}\phi_{;r}^*\phi_{;r} + m^2|\phi|^2) \quad (10)$$

up to here from note.tex

IV. INITIAL DATA

The spacetime line element, in its most general format, is

$$ds^2 = (-\alpha^2 + a^2\beta^2)dt^2 + 2a^2\beta dt dr + a^2 dr^2 + r^2 b^2 d\Omega^2 \quad (11)$$

To set up the initial data, we assume

$$a = b = \psi^2 \quad (12)$$

$$\beta = 0 \quad (13)$$

$$(14)$$

To set up ID in a 3d cartesian coordinate, We use the following form for the boson star initial data.

$$ds^2 = -\alpha^2 dt^2 + \psi^4 \delta_{ij} dx^i dx^j \quad (15)$$

We assume

$$\phi = \phi_0 e^{-i\omega t} \quad (16)$$

We will use G_{00} , Klein-Gordon Equation, and maximal slicing condition.(since we are gonna use maximal slicing condition for evolution, consider this when generating ID)(conformal flatness is only for ID and maximal condition fixes lapse function which we are free to choose-as far as it's consistent with other conditions.)

Then, we get in spherical coordinate system,

$$G_{00} = -4\frac{\alpha^2}{\psi^5 r}(r\psi_{,rr} + 2\psi_{,r}) \quad (17)$$

$$G_{rr} = \frac{1}{\alpha^2 r \psi^2}(2\alpha_{,r}^2 r \psi \psi_{,r} + 4r(\psi_{,r})^2 \alpha^2 + 4\psi_{,r} \alpha^2 \psi + \alpha_{,r}^2 \psi^2) \quad (18)$$

$$T_{00} = \frac{1}{2}(\omega^2 \phi^2 + \alpha^2 \psi^{-4}(\phi_{,r})^2 + \alpha^2 m^2 \phi^2 + \alpha^2 \frac{\lambda}{2} \phi^4) \quad (19)$$

$$T_{rr} = \frac{1}{2}((\phi_{,r})^2 + \psi^4 \frac{\omega^2}{\alpha^2} \phi^2 - \psi^4 m^2 \phi^2 - \psi^4 \frac{\lambda}{2} \phi^4) \quad (20)$$

$$g_{\mu\nu} = (-\alpha^2, \psi^4, \psi^4 r^2, \psi^4 r^2 \sin^2 \theta) \quad (21)$$

$$g^{\mu\nu} = (-\frac{1}{\alpha^2}, \psi^{-4}, \psi^{-4} r^{-2}, \psi^{-4} r^{-2} \sin^{-2} \theta) \quad (22)$$

$$(23)$$

Klein-Gordon equation:

$$(Box)\phi - m^2\phi - \lambda|\phi|^2\phi = 0 \quad (24)$$

$$\partial_\mu(\alpha\sqrt{h}g^{\mu\nu}\partial_\nu\phi) = \alpha\sqrt{h}(m^2\phi + \lambda|\phi|^2\phi) \quad (25)$$

$$\partial_r(\alpha\sqrt{h}g^{rr}\partial_r\phi) = \alpha\sqrt{h}(m^2 - \frac{\omega^2}{\alpha^2} + \lambda|\phi|^2)\phi \quad (26)$$

$$\sqrt{h} = \psi^6 r^2 \sin\theta \quad (27)$$

$$g^{rr} = \psi^{-4} \quad (28)$$

$$(\alpha r\psi^2)\partial_{rr}\phi + (2\alpha\psi^2 + \alpha_{,r}r\psi^2 + 2\alpha r\psi\psi_{,r})\phi_{,r} = \alpha\psi^6 r(m^2 - \frac{\omega^2}{\alpha^2} + \lambda|\phi|^2)\phi \quad (29)$$

May. 2nd, 1998

now as Matt suggested I will use maximal slicing throughout the evolution. therefore initial data should be consistent with this condition.

I will use three equations to set up spherically symmetric initial data. They are $G_{tt} = 8\pi T_{tt}$, Maximal slicing condition, Klein-Gordon equation.

$G_{tt} = 8\pi T_{tt}$ and Klein-Gordon equation(29) are given above.

First assumptions going into ID are

- (1) KG field ϕ takes the form $\phi(r, t) = \phi_0(r)e^{-i\omega t}$
- (2) time-symmetric data ie, $K_{ij} = 0$
- (3) conformally flat(15) condition(which is true only on initial slice)
- (4) zero shift, ie, $\beta^i = 0$

Now maximal slicing condition is given by

$$\alpha|_i^i = \alpha(K_{ij}K^{ij} + 4\pi(S + \rho)) \quad (30)$$

$$\alpha|_i^i = \frac{1}{\psi^6 r^2}(r^2\psi^2\alpha')' \quad (31)$$

$$\rho = \frac{1}{2}(\frac{\omega^2}{\alpha^2}\phi^2 + \frac{1}{\psi^4}(\phi_{,r})^2 + m^2\phi^2 + \frac{\lambda}{2}\phi^4) \quad (32)$$

$$S = tr(S_{ij}) = \frac{3}{2}(\frac{\omega^2}{\alpha^2} - m^2 - \frac{\lambda}{2}\phi^2)\phi^2 - \frac{1}{2}\frac{1}{\psi^4}(\phi_{,r})^2 \quad (33)$$

$$S + \rho = (2\frac{\omega^2}{\alpha^2} - m^2 - \frac{\lambda}{2}\phi^2)\phi^2 \quad (34)$$

(here i referred matt's thesis, page79 (4.7))

So maximal slicing condition is given by

$$\frac{1}{\psi^6 r^2}(r^2\psi^2\alpha')' = 4\pi\alpha(2\frac{\omega^2}{\alpha^2} - m^2 - \frac{\lambda}{2}\phi^2)\phi^2 \quad (35)$$

Let $A = \alpha'$. Then,

$$A' + (\frac{2}{r} + 2\frac{\psi'}{\psi})A - 4\pi\psi^4\alpha(2\frac{\omega^2}{\alpha^2} - m^2 - \frac{\lambda}{2}\phi^2)\phi^2 = 0 \quad (36)$$

So writing down the equations to solve in first order forms,

$$\psi' = \Psi \quad (37)$$

$$\Psi' = -\frac{2}{r}\Psi - 8\pi\frac{1}{8}(\psi\Phi^2 + \psi^5(\frac{\omega^2}{\alpha^2} + m^2 + \frac{\lambda}{2}\phi^2)\phi^2) \quad (38)$$

$$\phi' = \Phi \quad (39)$$

$$\Phi' = -(\frac{2}{r} + \frac{A}{\alpha} + \frac{2\Psi}{\psi})\Phi + \psi^4(m^2 - \frac{\omega^2}{\alpha^2} + \lambda\phi^2)\phi \quad (40)$$

$$\alpha' = A \quad (41)$$

$$A' = -(\frac{2}{r} + 2\frac{\Psi}{\psi})A + 4\pi\psi^4\alpha(2\frac{\omega^2}{\alpha^2} - m^2 - \frac{\lambda}{2}\phi^2)\phi^2 \quad (42)$$

Now, let me derive the equations for **harmonic** slicing condition.

$$\alpha = \psi^6 \quad (43)$$

Then

$$\psi' = \Psi \quad (44)$$

$$\Psi' = -\frac{2}{r}\Psi - 8\pi\frac{1}{8}(\psi\Phi^2 + \psi^5(\frac{\omega^2}{\psi^{12}} + m^2 + \frac{\lambda}{2}\phi^2)\phi^2) \quad (45)$$

$$\phi' = \Phi \quad (46)$$

$$\Phi' = -(\frac{2}{r} + \frac{8\Psi}{\psi})\Phi + \psi^4(m^2 - \frac{\omega^2}{\psi^{12}} + \lambda\phi^2)\phi \quad (47)$$

Stable branch lies below $\phi_0 = 0.0382$. This is from PRD **42** Seidel and Suen paper. They have $\Phi = \frac{\phi}{\sqrt{4\pi G(=1)}}$ where Φ is our(Dale's) scalar variable and maximum stable ϕ is given by 0.271 in that paper. So maximum kg field at the origin for us is 0.0764. Any initial data with central density above the value should be physically as well as numerically unstable.

ADM mass defined for conformal metric: $m = -\frac{1}{2\pi} \int \nabla^2 \psi d^3x$

In spherically symmetric spacetime, $m = \lim_{r \rightarrow \infty} (-2r^2 \psi_{,r})$ (from matt's thesis.)

To generate initial data:

$$\phi(t, r) = \phi_0(r) e^{-i\omega t} \quad (48)$$

Initial data generator gives us $\phi_0(r)$. Then,

$$\phi_{,x} = \frac{\partial \phi_0(r)}{\partial r} \frac{x}{r} \quad (49)$$

$$\phi_{,y} = \frac{\partial \phi_0(r)}{\partial r} \frac{y}{r} \quad (50)$$

$$\phi_{,z} = \frac{\partial \phi_0(r)}{\partial r} \frac{z}{r} \quad (51)$$

$$\phi_{,t} = -i\omega \phi_0(r) \quad (52)$$

Dimensional argument for mass of boson star. $m_{PI} = G^{-1/2}$, $G = m_{PI}^{-2}$ Schwarzschild radius $2MG = r$ where $r \sim \frac{1}{p}$ (p is momentum) from the Heizenberg uncertainty principle. $p \approx m$ for relativistic regime. Therefore $M = \frac{r}{2G} \sim \frac{1/m}{2/M_{PI}^2} \sim \frac{M_{PI}^2}{m}$.

V. OUTER BOUNDARY CONDITION

(1) scalar field: outgoing wave condition(robin type)

To be rigorous, we have to adopt pretty complicated outer boundary condition. For massless case, the propagation speed is independent of the wavelength (because $\frac{\omega}{k}$ is constant asymptotically) so the outgoing wave condition is simply given by

$$\frac{1}{\alpha} \frac{\partial(r\phi)}{\partial t} + \frac{1}{g_{rr}} \frac{\partial(r\phi)}{\partial r} = 0 \quad (53)$$

For massive case, you use $\omega^2 = k^2 + m^2$ for SR(Newtonian) case. (We get this from the KG eqn. in SR limit assuming that $\phi = \Sigma_k e^{i(\omega t - \mathbf{p} \cdot \mathbf{x})}$) But for massive GR case, the dispersion relation is $(\omega/\alpha)^2 = (k/g_{rr})^2 + m^2$.

But for the starter, I will implement a simplest possible OBC. Which is *flat* space *massless* outgoing radiation condition(Matt thesis p68). Which is given by

$$(r\phi)_{,t} + (r\phi)_{,r} = 0 \quad (54)$$

at a large r .

$$(r\phi)_{,t} + (r\phi)_{,r} = 0 \quad (55)$$

$$r\phi_{,t} + \phi + r\phi_{,r} = 0 \quad (56)$$

Now we have

$$\phi_{,x} = \phi_{,r} \frac{\partial r}{\partial x} \quad (57)$$

$$\phi_{,y} = \phi_{,r} \frac{\partial r}{\partial y} \quad (58)$$

$$\phi_{,z} = \phi_{,r} \frac{\partial r}{\partial z} \quad (59)$$

So we get after multiplying by $\frac{\partial r}{\partial x}$, etc..

$$x\phi_{,t} + \frac{x}{r}\phi + r\phi_{,x} = 0 \quad (60)$$

$$y\phi_{,t} + \frac{y}{r}\phi + r\phi_{,y} = 0 \quad (61)$$

$$z\phi_{,t} + \frac{z}{r}\phi + r\phi_{,z} = 0 \quad (62)$$

Apply this condition at the $n + \frac{1}{2}$ level.

$$\phi_{,t} = \frac{\phi_{ijk}^{n+1} - \phi_{ijk}^n}{\Delta t} \quad (63)$$

For $x_i = x_{min}$

$$\left(\frac{\partial \phi}{\partial x}\right)_{ijk} = \frac{1}{2\Delta x}(-\phi_{i+2,jk} + 4\phi_{i+1,jk} - 3\phi_{ijk}) \quad (64)$$

For $x_i = x_{max}$

$$\left(\frac{\partial \phi}{\partial x}\right)_{ijk} = \frac{-1}{2\Delta x}(-\phi_{i-2,jk} + 4\phi_{i-1,jk} - 3\phi_{ijk}) \quad (65)$$

Same for y and $z...$

$$r = \sqrt{x^2 + y^2 + z^2}.$$

At $x = x_{min}$,

$$\phi_{ijk}^{n+1} = \phi_{ijk}^n - \frac{\Delta t}{x_{min}} \left(\frac{x_{min}}{r} \phi_{ijk}^{n+1} + \frac{r}{2\Delta x} (-\phi_{i+2,jk}^{n+1} + 4\phi_{i+1,jk}^{n+1} - 3\phi_{ijk}^{n+1}) \right) \quad (66)$$

$$\left(1 + \frac{\Delta t}{r} - \frac{3r\Delta t}{2x_{min}\Delta x}\right) \phi_{ijk}^{n+1} = \phi_{ijk}^n - \frac{r\Delta t}{2x_{min}\Delta x} (-\phi_{i+2,jk}^{n+1} + 4\phi_{i+1,jk}^{n+1}) \quad (67)$$

At $x = x_{max}$,

$$\phi_{ijk}^{n+1} = \phi_{ijk}^n - \frac{\Delta t}{x_{max}} \left(\frac{x_{max}}{r} \phi_{ijk}^{n+1} + \frac{-r}{2\Delta x} (-\phi_{i-2,jk}^{n+1} + 4\phi_{i-1,jk}^{n+1} - 3\phi_{ijk}^{n+1}) \right) \quad (68)$$

$$\left(1 + \frac{\Delta t}{r} + \frac{3r\Delta t}{2x_{max}\Delta x}\right) \phi_{ijk}^{n+1} = \phi_{ijk}^n + \frac{r\Delta t}{2x_{max}\Delta x} (-\phi_{i-2,jk}^{n+1} + 4\phi_{i-1,jk}^{n+1}) \quad (69)$$

Similar for y and $z....$

At large r ,

$$\phi_{,t} \sim \Pi \quad (70)$$

$$x\Pi_{,t} + \frac{x}{r}\Pi + r\Pi_{,x} = 0 \quad (71)$$

$$y\Pi_{,t} + \frac{y}{r}\Pi + r\Pi_{,y} = 0 \quad (72)$$

$$z\Pi_{,t} + \frac{z}{r}\Pi + r\Pi_{,z} = 0 \quad (73)$$

Conditions for spatial derivatives:

FOR Φ_x ,

$$x\Phi_{x,t} + r\Phi_{x,x} + \frac{2x}{r}\Phi_x + \Pi + \left(\frac{1}{r} - \frac{x^2}{r^3}\right)\phi = 0 \quad (74)$$

$$y\Phi_{x,t} + r\Phi_{x,y} - \frac{2xy}{r^3}\phi + \frac{y}{r}\Phi_x - \frac{xy}{r^2}\Pi = 0 \quad (75)$$

$$z\Phi_{x,t} + r\Phi_{x,z} - \frac{2xz}{r^3}\phi + \frac{z}{r}\Phi_x - \frac{xz}{r^2}\Pi = 0 \quad (76)$$

For $x = x_{min}$,

$$x_{min}(\Phi_{x,ijk}^{n+1} - \Phi_{x,ijk}^n) = -\Delta t \left(r \frac{1}{2\Delta x} (-\Phi_{x,i+2,jk}^{n+1} + 4\Phi_{x,i+1,jk}^{n+1} - 3\Phi_{x,ijk}^{n+1}) + \frac{2x_{min}}{r}\Phi_{x,ijk}^{n+1} + \Pi_{ijk}^{n+1} + \left(\frac{1}{r} - \frac{x_{min}^2}{r^3}\right)\phi_{ijk}^{n+1} \right) \quad (77)$$

$$(x_{min} + \frac{2x_{min}\Delta t}{r} - \frac{3r\Delta t}{2\Delta x})\Phi_{x,ijk}^{n+1} = x_{min}\Phi_{x,ijk}^n - \Delta t \left(r \frac{1}{2\Delta x} (-\Phi_{x,i+2,jk}^{n+1} + 4\Phi_{x,i+1,jk}^{n+1}) + \Pi_{ijk}^{n+1} + \left(\frac{1}{r} - \frac{x_{min}^2}{r^3}\right)\phi_{ijk}^{n+1} \right) \quad (78)$$

For $x = x_{max}$,

$$x_{max}(\Phi_{x,ijk}^{n+1} - \Phi_{x,ijk}^n) = -\Delta t \left(r \frac{-1}{2\Delta x} (-\Phi_{x,i-2,jk}^{n+1} + 4\Phi_{x,i-1,jk}^{n+1} - 3\Phi_{x,ijk}^{n+1}) + \frac{2x_{max}}{r}\Phi_{x,ijk}^{n+1} + \Pi_{ijk}^{n+1} + \left(\frac{1}{r} - \frac{x_{max}^2}{r^3}\right)\phi_{ijk}^{n+1} \right) \quad (79)$$

$$(x_{max} + \frac{2x_{max}\Delta t}{r} + \frac{3r\Delta t}{2\Delta x})\Phi_{x,ijk}^{n+1} = x_{max}\Phi_{x,ijk}^n - \Delta t \left(r \frac{-1}{2\Delta x} (-\Phi_{x,i-2,jk}^{n+1} + 4\Phi_{x,i-1,jk}^{n+1}) + \Pi_{ijk}^{n+1} + \left(\frac{1}{r} - \frac{x_{max}^2}{r^3}\right)\phi_{ijk}^{n+1} \right) \quad (80)$$

For $y = y_{min}$,

$$y_{min}(\Phi_{x,ijk}^{n+1} - \Phi_{x,ijk}^n) = -\Delta t \left(r \frac{1}{2\Delta y} (-\Phi_{x,i,j+2,k}^{n+1} + 4\Phi_{x,i,j+1,k}^{n+1} - 3\Phi_{x,ijk}^{n+1}) - \frac{2xy_{min}}{r^3}\phi_{ijk}^{n+1} + \frac{y_{min}}{r}\Phi_{x,ijk}^{n+1} - \frac{xy_{min}}{r^2}\Pi_{ijk}^{n+1} \right) \quad (81)$$

$$(y_{min} + \Delta t \frac{y_{min}}{r} - \frac{3r\Delta t}{2\Delta y})\Phi_{x,ijk}^{n+1} = y_{min}\Phi_{x,ijk}^n - \Delta t \left(r \frac{1}{2\Delta y} (-\Phi_{x,i,j+2,k}^{n+1} + 4\Phi_{x,i,j+1,k}^{n+1}) - \frac{2xy_{min}}{r^3}\phi_{ijk}^{n+1} - \frac{xy_{min}}{r^2}\Pi_{ijk}^{n+1} \right) \quad (82)$$

For $y = y_{max}$,

$$y_{max}(\Phi_{x,ijk}^{n+1} - \Phi_{x,ijk}^n) = -\Delta t \left(r \frac{-1}{2\Delta y} (-\Phi_{x,i,j-2,k}^{n+1} + 4\Phi_{x,i,j-1,k}^{n+1} - 3\Phi_{x,ijk}^{n+1}) - \frac{2xy_{max}}{r^3}\phi_{ijk}^{n+1} + \frac{y_{max}}{r}\Phi_{x,ijk}^{n+1} - \frac{xy_{max}}{r^2}\Pi_{ijk}^{n+1} \right) \quad (83)$$

$$(y_{max} + \Delta t \frac{y_{max}}{r} + \frac{3r\Delta t}{2\Delta y})\Phi_{x,ijk}^{n+1} = y_{max}\Phi_{x,ijk}^n - \Delta t \left(r \frac{-1}{2\Delta y} (-\Phi_{x,i,j-2,k}^{n+1} + 4\Phi_{x,i,j-1,k}^{n+1}) - \frac{2xy_{max}}{r^3}\phi_{ijk}^{n+1} - \frac{xy_{max}}{r^2}\Pi_{ijk}^{n+1} \right) \quad (84)$$

For $z = z_{min}$,

$$z_{min}(\Phi_{x,ijk}^{n+1} - \Phi_{x,ijk}^n) = -\Delta t \left(r \frac{1}{2\Delta z} (-\Phi_{x,i,j+2,k}^{n+1} + 4\Phi_{x,i,j+1,k}^{n+1} - 3\Phi_{x,ijk}^{n+1}) - \frac{2xz_{min}}{r^3}\phi_{ijk}^{n+1} + \frac{z_{min}}{r}\Phi_{x,ijk}^{n+1} - \frac{xz_{min}}{r^2}\Pi_{ijk}^{n+1} \right) \quad (85)$$

For $z = z_{max}$,

$$z_{max}(\Phi_{x,ijk}^{n+1} - \Phi_{x,ijk}^n) = -\Delta t \left(r \frac{-1}{2\Delta z} (-\Phi_{x,i,j-2,k}^{n+1} + 4\Phi_{x,i,j-1,k}^{n+1} - 3\Phi_{x,ijk}^{n+1}) - \frac{2xz_{max}}{r^3}\phi_{ijk}^{n+1} + \frac{z_{max}}{r}\Phi_{x,ijk}^{n+1} - \frac{xz_{max}}{r^2}\Pi_{ijk}^{n+1} \right) \quad (86)$$

FOR Φ_y ,

$$y\Phi_{y,t} + r\Phi_{y,y} + \frac{2y}{r}\Phi_y + \Pi + \left(\frac{1}{r} - \frac{y^2}{r^3}\right)\phi = 0 \quad (87)$$

$$z\Phi_{y,t} + r\Phi_{y,z} - \frac{2yz}{r^3}\phi + \frac{z}{r}\Phi_y - \frac{yz}{r^2}\Pi = 0 \quad (88)$$

$$x\Phi_{y,t} + r\Phi_{y,x} - \frac{2yx}{r^3}\phi + \frac{x}{r}\Phi_y - \frac{yx}{r^2}\Pi = 0 \quad (89)$$

FOR Φ_z ,

$$z\Phi_{z,t} + r\Phi_{z,z} + \frac{2z}{r}\Phi_z + \Pi + \left(\frac{1}{r} - \frac{z^2}{r^3}\right)\phi = 0 \quad (90)$$

$$x\Phi_{z,t} + r\Phi_{z,x} - \frac{2zx}{r^3}\phi + \frac{x}{r}\Phi_z - \frac{zx}{r^2}\Pi = 0 \quad (91)$$

$$y\Phi_{z,t} + r\Phi_{z,y} - \frac{2zy}{r^3}\phi + \frac{y}{r}\Phi_z - \frac{zy}{r^2}\Pi = 0 \quad (92)$$

Full 3d Einstein Klein Gordon equation.

Stress energy tensor $T^{\mu\nu}$ is given by

$$T^{\mu\nu} = \frac{1}{2}(\phi^{*,\mu}\phi^{,\nu} + \phi^{,\mu}\phi^{*,\nu}) - \frac{1}{2}g^{\mu\nu}(\phi_{,\lambda}^*\phi^{,\lambda} + m^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4) \quad (93)$$

The Metric is given in the following form at all times.

$$ds^2 = -(\alpha^2 - \beta_i\beta^i)dt^2 + 2\beta_idx^i dt + g_{ij}dx^i dx^j \quad (94)$$

$$\rho = T^{\mu\nu}n_\mu n_\nu \quad (95)$$

$$j^i = -n_\mu T^{\mu i} \quad (96)$$

$$S_{ij} = T_{ij} \quad (97)$$

$$\quad (98)$$

Auxiliary variables will be used.

$$\Phi_i = \phi_{,i} \quad (99)$$

$$\Pi = \frac{\sqrt{h}}{\alpha}(\phi_{,t} - \beta^i\phi_{,i}) \quad (100)$$

$$\phi_{,t} = \frac{\alpha}{\sqrt{h}}\Pi + \beta^i\Phi_i \quad (101)$$

where $n_\mu = (-\alpha, 0, 0, 0)$ and $g^{\mu\nu} = \begin{pmatrix} -\frac{1}{\alpha^2} & \frac{\beta^i}{\alpha^2} \\ \frac{\beta^i}{\alpha^2} & g^{ij} - \frac{\beta^i\beta^j}{\alpha^2} \end{pmatrix}$

so $n^\mu = g^{\mu\nu}n_\nu = (\frac{1}{\alpha}, -\frac{\beta^i}{\alpha})$. n^μ is a timelike Killing vector for zero shift so in that case ρ should be conserved quantity.(prove this, i.e., $T^{\mu\nu}n_\mu n_\nu$ is constant along timelike geodesics if n_μ is timelike Killing vector.)

Geometrical (physical) variable T is $G = -R = 8\pi T$. $T = g^{\mu\nu}T_{\mu\nu} = -\frac{1}{\alpha^2}T_{00} + g^{ij}T_{ij} = -\rho + S$ (with $\beta^i = 0$).

$$\phi^{,t} = \phi^{;t} = g^{t\mu}\phi_{,\mu} = g^{t\mu}\phi_{,\mu} = -\frac{1}{\alpha^2}\phi_{,t} + \frac{\beta^i}{\alpha^2}\phi_{,i} = -\frac{1}{\alpha\sqrt{h}}\Pi \quad (102)$$

$$\phi^{,i} = \phi^{;i} = g^{i\mu}\phi_{,\mu} = g^{i\mu}\phi_{,\mu} = \frac{\beta^i}{\alpha^2}\phi_{,t} + ({}^{(3)}g^{ij} - \frac{\beta^i\beta^j}{\alpha^2})\phi_{,j} = \frac{\beta^i}{\alpha\sqrt{h}}\Pi + {}^{(3)}g^{ij}\Phi_j \quad (103)$$

$$\phi^{*,t}\phi^{,t} = \frac{1}{\alpha^2 h}(\Pi_{re}^2 + \Pi_{im}^2) \quad (104)$$

$$\phi^{*,t}\phi^{,i} + \phi^{,t}\phi^{*,i} = -\frac{2\beta^i}{\alpha^2 h}(\Pi_{re}^2 + \Pi_{im}^2) - \frac{2({}^{(3)}g^{ij})}{\alpha\sqrt{h}}(\Pi_{re}\Phi_{j,re} + \Pi_{im}\Phi_{j,im}) \quad (105)$$

$$\begin{aligned} \phi_{,\lambda}^*\phi^{,\lambda} &= \phi_{,t}^*\phi^{,t} + \phi_{,i}^*\phi^{,i} = (\frac{\alpha}{\sqrt{h}}\Pi^* + \beta^i\Phi_i^*)(-\frac{1}{\alpha\sqrt{h}}\Pi) + \Phi_i^*(-\frac{\beta^i}{\alpha\sqrt{h}}\Pi + {}^{(3)}g^{ij}\Phi_j) \\ &= -\frac{1}{h}(\Pi_{re}^2 + \Pi_{im}^2) + {}^{(3)}g^{ij}(\Phi_{i,re}\Phi_{j,re} + \Phi_{i,im}\Phi_{j,im}) \end{aligned} \quad (106)$$

$$\phi_{,i}^*\phi_{,j} + \phi_{,i}\phi_{,j}^* = 2(\Phi_{i,re}\Phi_{j,re} + \Phi_{i,im}\Phi_{j,im}) \quad (107)$$

$$\rho = \alpha^2\phi^{*,t}\phi^{,t} + \frac{1}{2}(\phi_{,\lambda}^*\phi^{,\lambda} + m^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4) \quad (108)$$

$$j^i = \frac{\alpha}{2}(\phi^{*,t}\phi^{,i} + \phi^{,t}\phi^{*,i} - g^{ti}(\phi_{,\lambda}^*\phi^{,\lambda} + m^2|\phi|^2) + \frac{\lambda}{2}|\phi|^4) \quad (109)$$

$$S_{ij} = \frac{1}{2}(\phi_{,i}^*\phi_{,j} + \phi_{,i}\phi_{,j}^*) - \frac{1}{2}g_{ij}(\phi_{,\lambda}^*\phi^{,\lambda} + m^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4) \quad (110)$$

$$trS = -\frac{3}{2}\phi_{,t}^*\phi^{,t} - \frac{1}{2}\phi_{,i}^*\phi^{,i} - \frac{3}{2}(m^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4) \quad (111)$$

$$\rho = \frac{1}{2} \frac{1}{h} (\Pi_{re}^2 + \Pi_{im}^2) + \frac{1}{2} {}^{(3)}g^{ij} (\Phi_{i,re} \Phi_{j,re} + \Phi_{i,im} \Phi_{j,im}) + \frac{1}{2} m^2 (\phi_{re}^2 + \phi_{im}^2) + \frac{\lambda}{4} (\phi_{re}^2 + \phi_{im}^2)^2 \quad (112)$$

$$j^i = -\frac{\beta^i}{2\alpha h} (\Pi_{re}^2 + \Pi_{im}^2) - \frac{{}^{(3)}g^{ij}}{\sqrt{h}} (\Pi_{re} \Phi_{j,re} + \Pi_{im} \Phi_{j,im}) - \frac{\beta^i}{2\alpha} {}^{(3)}g^{kl} (\Phi_{k,re} \Phi_{l,re} + \Phi_{k,im} \Phi_{l,im}) - \frac{\beta^i}{2} m^2 (\phi_{re}^2 + \phi_{im}^2) - \frac{\beta^i \lambda}{2} (\phi_{re}^2 + \phi_{im}^2)^2 \quad (113)$$

$$S_{ij} = \Phi_{i,re} \Phi_{j,re} + \Phi_{i,im} \Phi_{j,im} - \frac{1}{2} g_{ij} \left(-\frac{1}{h} (\Pi_{re}^2 + \Pi_{im}^2) + {}^{(3)}g^{kl} (\Phi_{k,re} \Phi_{l,re} + \Phi_{k,im} \Phi_{l,im}) + m^2 (\phi_{re}^2 + \phi_{im}^2) + \frac{\lambda}{2} (\phi_{re}^2 + \phi_{im}^2)^2 \right) \quad (114)$$

3+1 form of Klein Gordon equation using causal differencing.

$$\Phi_i = \frac{\partial \phi}{\partial x_i} \quad (115)$$

$$\Pi = \frac{\sqrt{h}}{\alpha} (\partial_t - \beta^i \partial_i) \phi \quad (116)$$

$$\Pi_{,t} - \beta^i \Pi_{,i} = \beta_{,i}^i \Pi + (\alpha \sqrt{h} h^{ij} \Phi_{j,i} - \alpha \sqrt{h} m^2 \phi - \alpha \sqrt{h} \lambda |\phi|^2 \phi) \quad (117)$$

$$\Phi_{i,t} - \beta^j \Phi_{j,i} = \left(\frac{\alpha}{\sqrt{h}} \Pi \right)_{,i} + \beta_{,i}^j \Phi_j \quad (118)$$

$$\phi_{,t} - \beta^i \phi_{,i} = \frac{\alpha}{\sqrt{h}} \Pi \quad (119)$$

Derivation of Maximal Slicing condition

$$\partial_t K_{ij} = -D_i D_j \alpha + \alpha (R_{ij} + K K_{ij} - 2K_{il} K_j^l) + \mathbb{L}_\beta K_{ij} - 8\pi \alpha (S_{ij} - \frac{1}{2} g_{ij} (S - \rho)) \quad (120)$$

Taking trace,

$$\partial_t K = -D^i D_i \alpha + \alpha (R + K K - 2K_{il} K^{li}) - 4\pi \alpha (-S + 3\rho) + g^{ij} \mathbb{L}_\beta K_{ij} + (\partial_t g^{ij}) K_{ij} \quad (121)$$

Using the Hamiltonian constraint equation,

$$R + K^2 - K_{ij} K^{ij} = 16\pi \rho \quad (122)$$

we get,

$$\partial_t K = -D^i D_i \alpha + \alpha (-K_{il} K^{li}) + 4\pi \alpha (S + \rho) + g^{ij} \mathbb{L}_\beta K_{ij} + (\partial_t g^{ij}) K_{ij} \quad (123)$$

$$g^{ij} \mathbb{L}_\beta K_{ij} + (\partial_t g^{ij}) K_{ij} = \mathbb{L}_\beta K - K_{ij} \mathbb{L}_\beta g^{ij} + K_{ij} \partial_t g^{ij} = \mathbb{L}_\beta K + (\partial_t g^{ij} - \mathbb{L}_\beta g^{ij}) K_{ij} \quad (124)$$

$$\partial_t g_{ij} = -2\alpha K_{ij} + \mathbb{L}_\beta g_{ij} \quad (125)$$

$$(\partial_t - \mathbb{L}_\beta) g_{ij} = -2\alpha K_{ij} \quad (126)$$

Then

$$g^{ki} g^{lj} (\partial_t - \mathbb{L}_\beta) g_{ij} = -2\alpha K^{kl} \quad (127)$$

$$(\partial_t - \mathbb{L}_\beta) g^{kl} - g^{ki} g_{ij} (\partial_t - \mathbb{L}_\beta) g^{lj} - g^{lj} g_{ij} (\partial_t - \mathbb{L}_\beta) g^{ki} = -2\alpha K^{kl} \quad (128)$$

$$(\partial_t - \mathbb{L}_\beta) g^{kl} - \delta_j^k (\partial_t - \mathbb{L}_\beta) g^{lj} - \delta_j^l (\partial_t - \mathbb{L}_\beta) g^{kj} = -2\alpha K^{kl} \quad (129)$$

$$(\partial_t - \mathbb{L}_\beta) g^{kl} = 2\alpha K^{kl} \quad (130)$$

$$\begin{aligned} \partial_t K &= -D^i D_i \alpha + \alpha (-K_{il} K^{li}) + 4\pi \alpha (S + \rho) + \mathbb{L}_\beta K + 2\alpha K^{ij} K_{ij} \\ &= -D^i D_i \alpha + \alpha (K_{ij} K^{ij} + 4\pi (S + \rho)) + \mathbb{L}_\beta K \end{aligned} \quad (131)$$

Maximal slicing condition requires $K = 0$,

$$D^i D_i \alpha = \alpha(K_{ij} K^{ij} + 4\pi(S + \rho)) \quad (132)$$

$$D^i D_i \alpha = g^{ij} \partial_i \partial_j \alpha - g^{ij} \Gamma_{ij}^k \partial_k \alpha \quad (133)$$

Then, Maximal Slicing condition is given by

$$g^{ij} \partial_i \partial_j \alpha - g^{ij} \Gamma_{ij}^k \partial_k \alpha = \alpha(K_{ij} K^{ij} + 4\pi(S + \rho)) \quad (134)$$

$$g^{ij} \Gamma_{ij}^k = g^{ij} \frac{1}{2} g^{kl} (g_{jl,i} + g_{il,j} - g_{ij,l}) = g^{ij} g^{kl} g_{jl,i} - \frac{1}{2} g^{ij} g^{kl} g_{ij,l} \quad (135)$$

Using identities

$$g^{\nu\alpha} g_{\nu\alpha,\mu} = \frac{g_{,\mu}}{g} \quad (136)$$

$$g^{kl} g_{jl,i} = -g_{,i}^{kl} g_{jl} \quad (137)$$

we get,

$$g^{ij} \Gamma_{ij}^k = -g_{,i}^{ki} - \frac{1}{2} g^{kl} \frac{g_{,l}}{g} = \frac{1}{2} (g_{,i}^{ki} - g^{ij} (g^{kl} g_{ij})_{,l}) \quad (138)$$

Finally, Maximal slicing condition is

$$g^{ij} \partial_i \partial_j \alpha + (g_{,i}^{ki} + \frac{1}{2} g^{kl} \frac{g_{,l}}{g}) \partial_k \alpha - (K_{ij} K^{ij} + 4\pi(S + \rho)) \alpha = 0 \quad (139)$$

VI. MAXIMAL SLICING CONDITION: WROTE MY OWN MG SOLVERS FOR IT

$$g^{ij} \partial_i \partial_j \alpha - g^{ij} \Gamma_{ij}^k \partial_k \alpha = \alpha(K_{ij} K^{ij} + 4\pi(S + \rho)) \quad (140)$$

Let $A^k = g^{ij} \Gamma_{ij}^k$ and $B = K_{ij} K^{ij} + 4\pi(S + \rho)$. Then we are to solve

$$g^{ij} \partial_i \partial_j \alpha - A^k \partial_k \alpha - B \alpha = 0 \quad (141)$$

$$(142)$$

(1) test with flat, no matter

(2) outer boundary for Maximal solver:robin

From [28] we know solving Eq.(140) is not enough to *control* K . K is drifting from zero and this cannot be controlled. Maximal solver doesn't put K back to zero when α is perturbed at any time. With lapse returned to the desirable value, but K ie. slicing still curved. I think that's because equation we solve implies $K_{,t} = 0$, not $K = 0$ at all times. History of lapse is important. They proposed driver condition to *actively* enforce maximal slicing condition. They proposed to solve

$$\frac{\partial K}{\partial t} + cK = 0 \quad (143)$$

$$D^i D_i \alpha - \alpha(K_{ij} K^{ij} + 4\pi(S + \rho)) = cK \quad (144)$$

They called this *K-driver*.

They use time evolution to solve elliptic equations. So,

$$\frac{\partial \alpha}{\partial t} = \epsilon(D^i D_i \alpha - \alpha(K_{ij} K^{ij} + 4\pi(S + \rho)) - cK) \quad (145)$$

Mon JUNE 30, 1997 and Fri OCT 03, 1997

Sensible and consistent slicing condition for boson star problem

- (1) maximal slicing, quasi-isotropic(isothermal) gauge
- (2) zero shift, isotropic
- (3) geodesic slicing

Isothermal gauge is a generalization of isotropic coordinates in spherically symmetric problems

Ref. Matt's Thesis

Bardeen and Piran, Physics Report **96**, pp205-250 (1983)

Petrich, Shapiro, and Teukolsky, PRD **31**, 2459 (1985)

A.M. Abrahams and C.R. Evans, PRD **37**, 318 (1988)

(NEW) how does isothermal gauge work out in the 3d Cartesian coord?

(NEW) so perhaps, the easiest choice would be geodesic slicing.

QI gauge line element expressed in terms of spherical-polar coordinates:

$$ds^2 = A^2(dr^2 + r^2 d\theta^2) + B^2 r^2 (\sin\theta d\phi + \xi d\theta)^2 \quad (146)$$

which stems from the three coordinate component conditions

$$g_{r\phi} = 0 \quad (147)$$

$$g_{r\theta} = 0 \quad (148)$$

$$g_{\theta\theta}g_{\phi\phi} - (g_{\theta\phi})^2 = g_{rr}g_{\phi\phi}r^2 \quad (149)$$

VII. EVOLUTION

Evolution Scheme for Boson Star code using CN iteration.

$$g^{n+1} = g^n, K^{n+1} = K^n, \phi^{n+1} = \phi^n \quad (150)$$

$$g^{n+\frac{1}{2}} = \frac{1}{2}(g^{n+1} + g^n), K^{n+\frac{1}{2}} = \frac{1}{2}(K^{n+1} + K^n), \phi^{n+\frac{1}{2}} = \frac{1}{2}(\phi^{n+1} + \phi^n) \quad (151)$$

```
do loop
  compute gdot, Kdot
  update  $g^{n+1}, K^{n+1}$ 
  get new  $g^{n+\frac{1}{2}}, K^{n+\frac{1}{2}}$ 
  compute kgRHS
  update  $\phi^{n+1}$ 
  get new  $\phi^{n+\frac{1}{2}}$ 
enddo
Causal differencing
```

VIII. WILSON AND MATTHEW

IGNORE below for now. It's relevant only for initial data where conformal flatness for metric is assumed.

Conformal flatness, compute ρ , j^i , and S^{ij} .

Physcial quantities are related to conformal quantities by

$$g_{ij} = \psi^4 \hat{g}_{ij} \quad (152)$$

$$g^{ij} = \psi^{-4} \hat{g}^{ij} \quad (153)$$

$$K^{ij} = \psi^{-10} \hat{K}^{ij} \quad (154)$$

$$K_{ij} = \psi^{-2} \hat{K}_{ij} \quad (155)$$

$$\rho = \psi^{-8} \hat{\rho} \quad (156)$$

$$j^i = \psi^{-10} \hat{j}^i \quad (157)$$

$$\hat{\alpha} = \psi \alpha \quad (158)$$

Assuming conformal flatness, from the definition of K_{ij} and the maximal slicing condition, we get

$$\dot{g}_{ij} = -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i \quad (159)$$

$$\hat{K}^{ij} = \frac{\psi^6}{2\alpha} (\hat{D}^i \beta^j + \hat{D}^j \beta^i - \frac{2}{3} \delta^{ij} \hat{D}_k \beta^k) \quad (160)$$

Maximal slicing condition ($\text{Tr } K = 0$) itself gives us a equation for α

$$\Delta \alpha - \alpha (K^{ij} K_{ij} + \frac{1}{2} (\rho + \text{tr} S)) = 0 \quad (161)$$

This equation can be written in terms of conformal quantities as

$$\hat{\Delta} \hat{\alpha} = \hat{\alpha} (\frac{7}{8} \psi^{-8} \hat{K}^{ij} \hat{K}_{ij} - \frac{1}{4} \hat{\rho} \psi^{-4} + \frac{1}{2} \psi^4 \text{tr} S) \quad (162)$$

Hamiltonian constraint equation gives us a equation for ψ

$$\hat{\Delta} \psi + \frac{1}{8} \hat{K}^{ij} \hat{K}_{ij} \psi^{-7} + \frac{1}{4} \hat{\rho} \psi^{-3} = 0 \quad (163)$$

Momentum constraint equations give us equations for β^i

$$\hat{D}_j \hat{K}^{ij} = \hat{j}^i \quad (164)$$

$$\hat{\Delta} \beta^j + \frac{1}{3} \hat{D}^j (\hat{D}_k \beta^k) = 2 \hat{K}^{ij} \hat{D}_j (\hat{\alpha} \psi^{-7}) + 2 \alpha \psi^{-6} \hat{j}^i \quad (165)$$

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Self-gravitating Massive scalar field can form boson stars (complex scalar) or oscillatons (real scalar)
(Astro)Physical motivation for the study of Boson star!!
*references on Boson Star on Kip. Thorne's paper Boson star configuration was introduced by Kaup and by Ruffini and Bonazzola in 1960 and current interest was sparked by Corpi, et al. who showed that, provided the scalar field has a self interaction, the boson star masses could be of the same order of magnitude as, or even much greater than, the Chandrasekhar mass.
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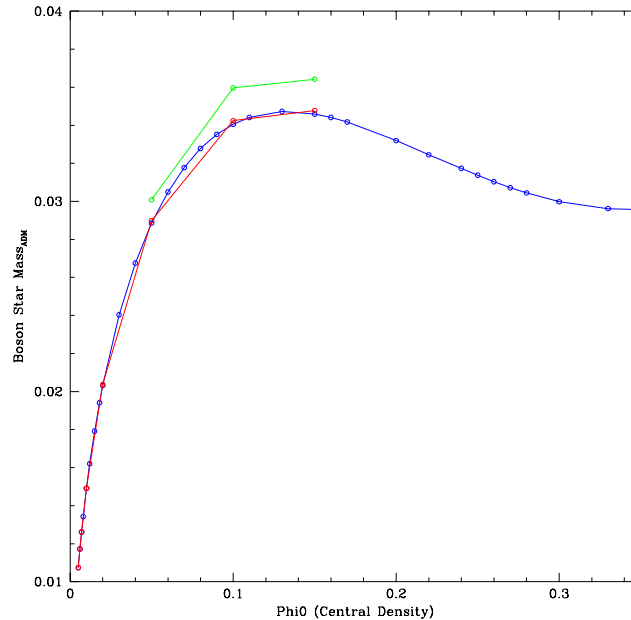


FIG. 1. Total mass .vs. central density, $\phi(r=0)$.

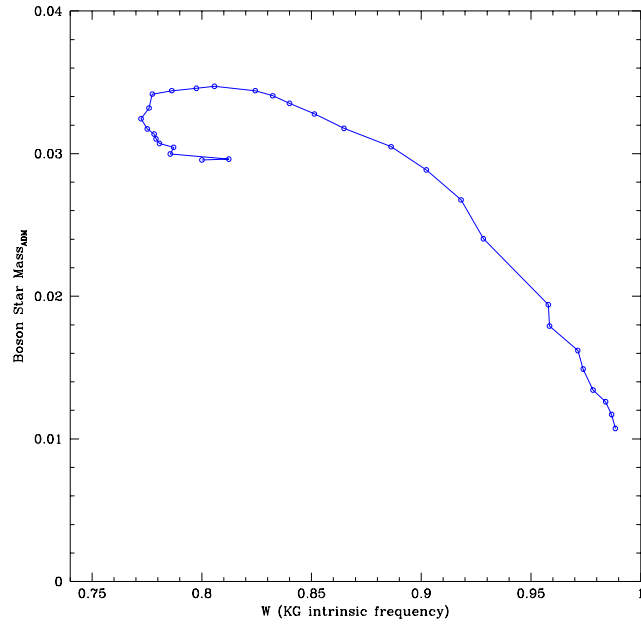


FIG. 2. Total mass .vs. W(KG intrinsic osc. freq.)